EUROPEAN CENTRAL BANK
EUROSYSTEM

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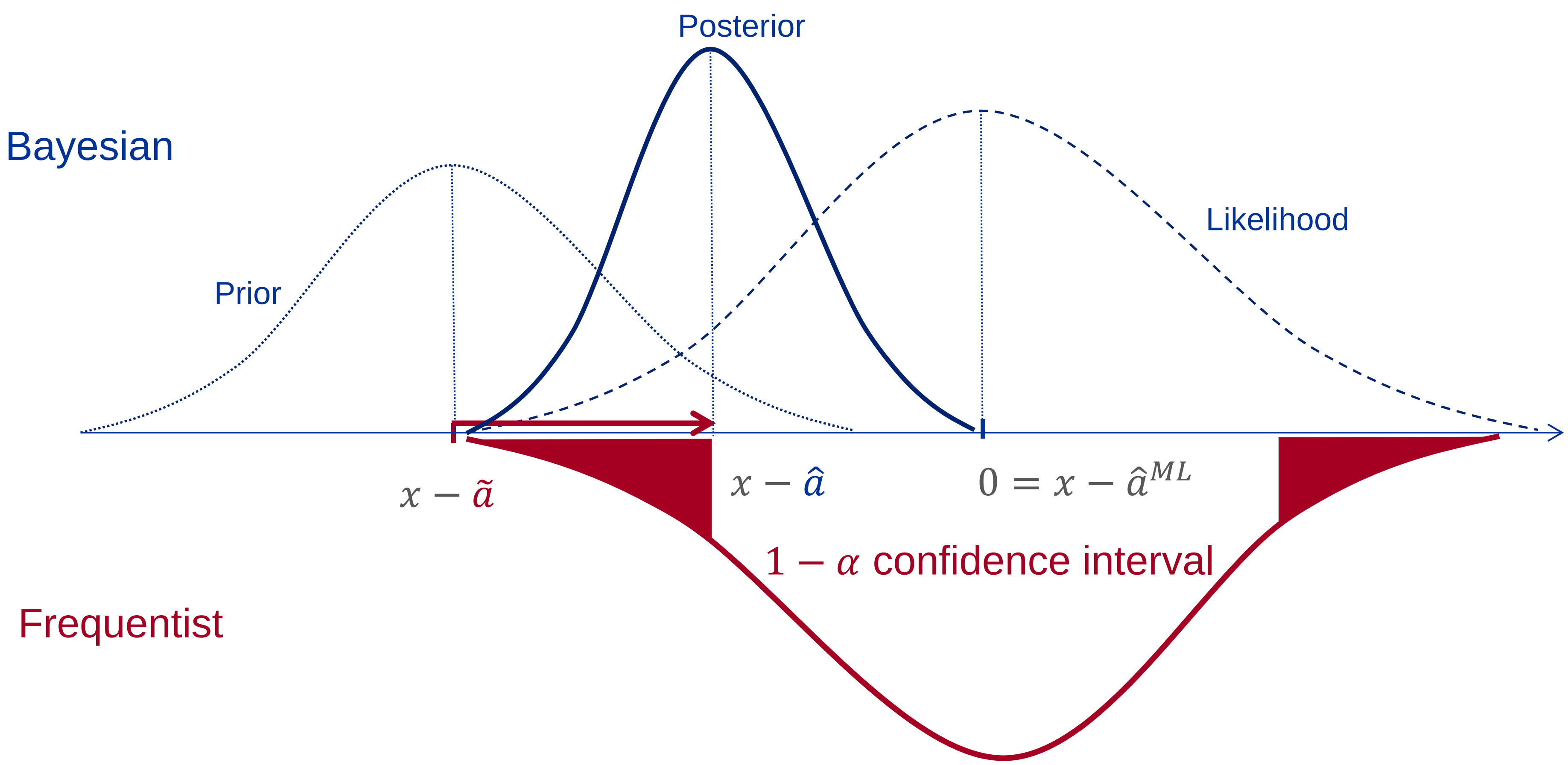
European Central Bank

Deciding with Judgment

May 2017

The decision rule incorporating judgment

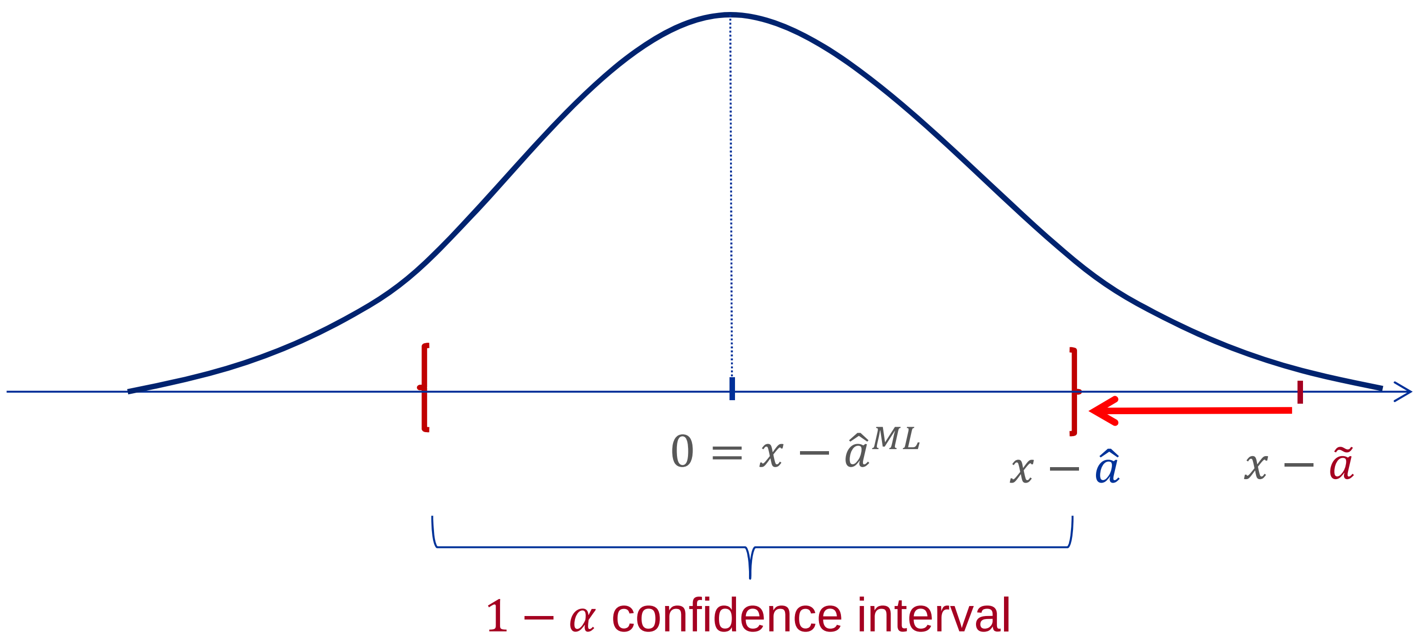
$X \sim N(\theta, 1)$ $\min_a (\theta - a)^2$ $X = x$



Prior $\rightarrow$ Judgment: $\{\tilde{a}, \alpha\}$

The decision rule incorporating judgment

$X \sim N(\theta, 1)$ $\min_a (\theta - a)^2$ $X = x$ Judgment: $\{\tilde{a}, \alpha\}$



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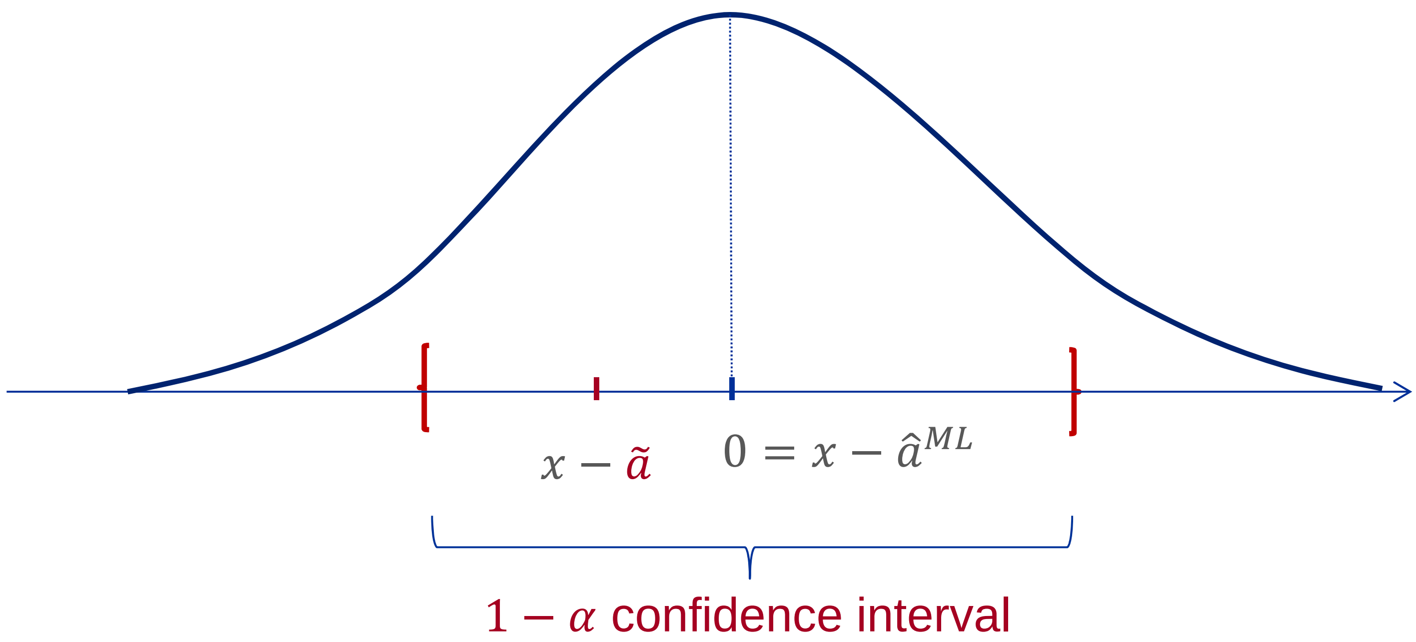
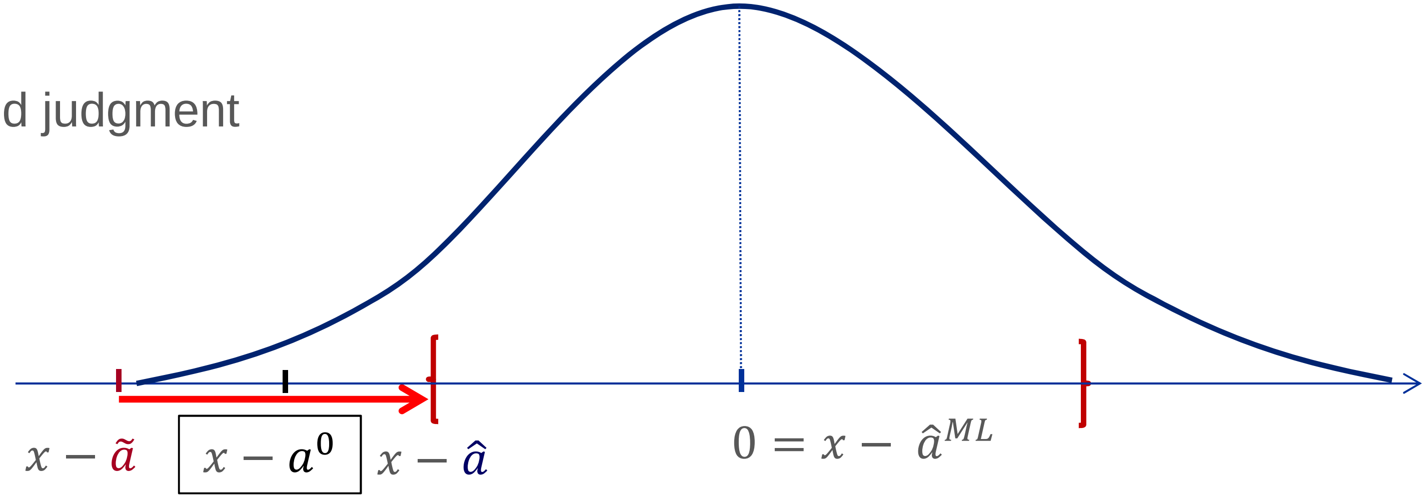


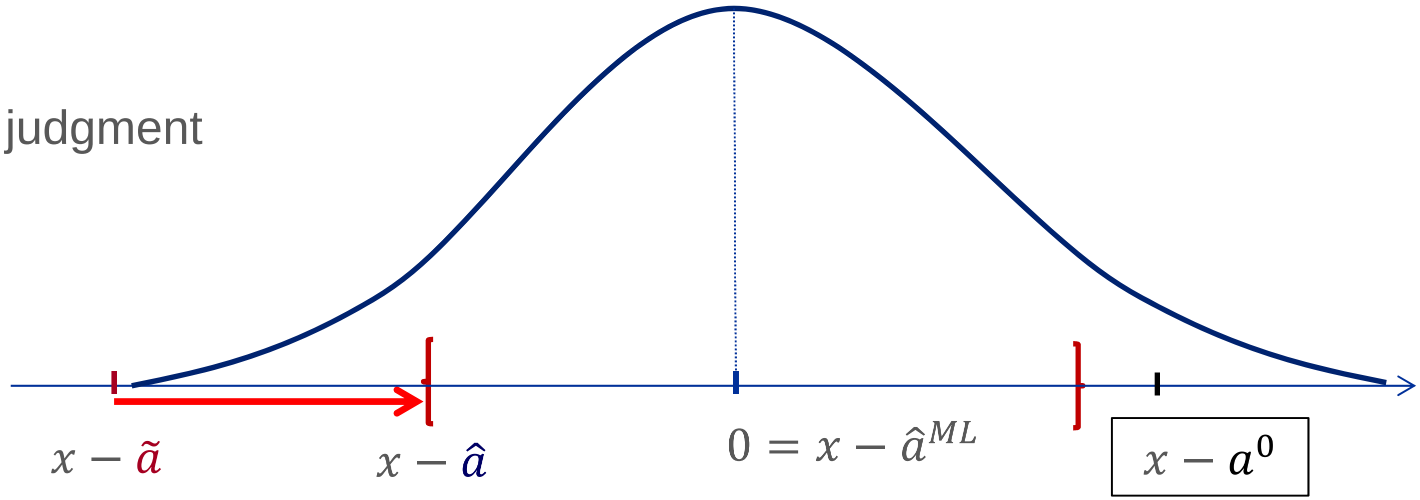
Clive Granger

"A good Bayesian is better than a non-Bayesian, who is better than a bad Bayesian."

Good judgment



Bad judgment



"Having good judgment is better than no judgement, which is better than bad judgment."

Outline

An asset allocation decision problem

Statistical decision rules

Equivalence between frequentist and Bayesian decision rules

Empirical illustration

The asset allocation problem – A working example

Investor holds cash, decides how much to invest in a stock index.

$X \sim N(\theta, 1)$ → return of stock index

a → share in stock index

$X = x$ → one available observation

Mean-variance loss function:

$$\min_a L(\theta, a) = -a\theta + 0.5a^2$$

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The Bayesian decision

If the decision maker knows her prior distribution $\mu(\theta)$, the Bayesian solution is:

$$\delta^{\mu}(x) = \arg\min_a \int L(\theta, a)\, d\mu(\theta|x)$$

Pro: Axiomatic foundation, Savage (1954)

Con: Prior is unknown

Neyman (1952): *"When a priori probabilities are not available (which Fisher presumed to be always the case and which I agree is almost always the case), then the formula of Bayes is not applicable."*

The frequentist decision

Key ingredients:

1. Judgment
2. Hypothesis testing
3. Confidence intervals

The frequentist decision: 1. Judgment

Judgment: $\mathrm{A} \equiv \{\tilde{a}, \alpha\}$

$\tilde{a}$: judgmental decision

α: confidence level

The frequentist decision: 2. Hypothesis testing

FOC: $\nabla_a L(\theta, a^0) = -\theta + a^0 = 0$

Evaluate FOC at:

$\theta \rightarrow \hat{\theta} = x$

$a^0 \rightarrow \tilde{a}$

Under H_0: $L(\theta, \tilde{a}) = 0$

$$-X + \tilde{a} \sim N(0,1)$$

Frequentist decision rule:

$$\begin{aligned}\delta^A(x) = \left(x + c_{\alpha/2}\right) \cdot I\left(-x + \tilde{a} < c_{\alpha/2}\right) + \\ + \tilde{a} \cdot I\left(c_{\alpha/2} \leq -x + \tilde{a} \leq c_{1-\alpha/2}\right) + \\ + \left(x + c_{1-\alpha/2}\right) \cdot I\left(-x + \tilde{a} > c_{1-\alpha/2}\right)\end{aligned}$$

The frequentist decision: 3. Confidence Intervals

$$\delta^A(x) = \left(x + c_{\alpha/2}\right) \cdot I\left(-x + \tilde{a} < c_{\alpha/2}\right) +$$
$$+ \tilde{a} \cdot I\left(c_{\alpha/2} \leq -x + \tilde{a} \leq c_{1-\alpha/2}\right) +$$
$$+ \left(x + c_{1-\alpha/2}\right) \cdot I\left(-x + \tilde{a} > c_{1-\alpha/2}\right)$$

describes the confidence interval of $-X + \tilde{a}$ under H_0: $L(\theta, \tilde{a}) = 0$

Economic interpretation of confidence intervals:

$P_\theta\left[L\left(\theta, \delta^A(X)\right) > L(\theta, \tilde{a})\right] < \alpha$

Economic interpretation of symmetric confidence intervals

They are the solution to minimax problem:

$$\min_{\delta} \max_{\theta} E_{\theta}[L(\theta, \delta^{A}(X))]$$

$$\text{s.t. } P_{\theta}\left[L\left(\theta, \delta^{A}(X)\right) > L(\theta, \tilde{a})\right] < \alpha$$

where

$$\delta^{A}(X) = \left(X + c_{\alpha/2}\right) \cdot I\left(-X + \tilde{a} < c_{\alpha/2}\right) + \\ + \tilde{a} \cdot I\left(c_{\alpha/2} \leq -X + \tilde{a} \leq c_{1-\alpha/2}\right) + \\ + \left(X + c_{1-\alpha/2}\right) \cdot I\left(-X + \tilde{a} > c_{1-\alpha/2}\right)$$

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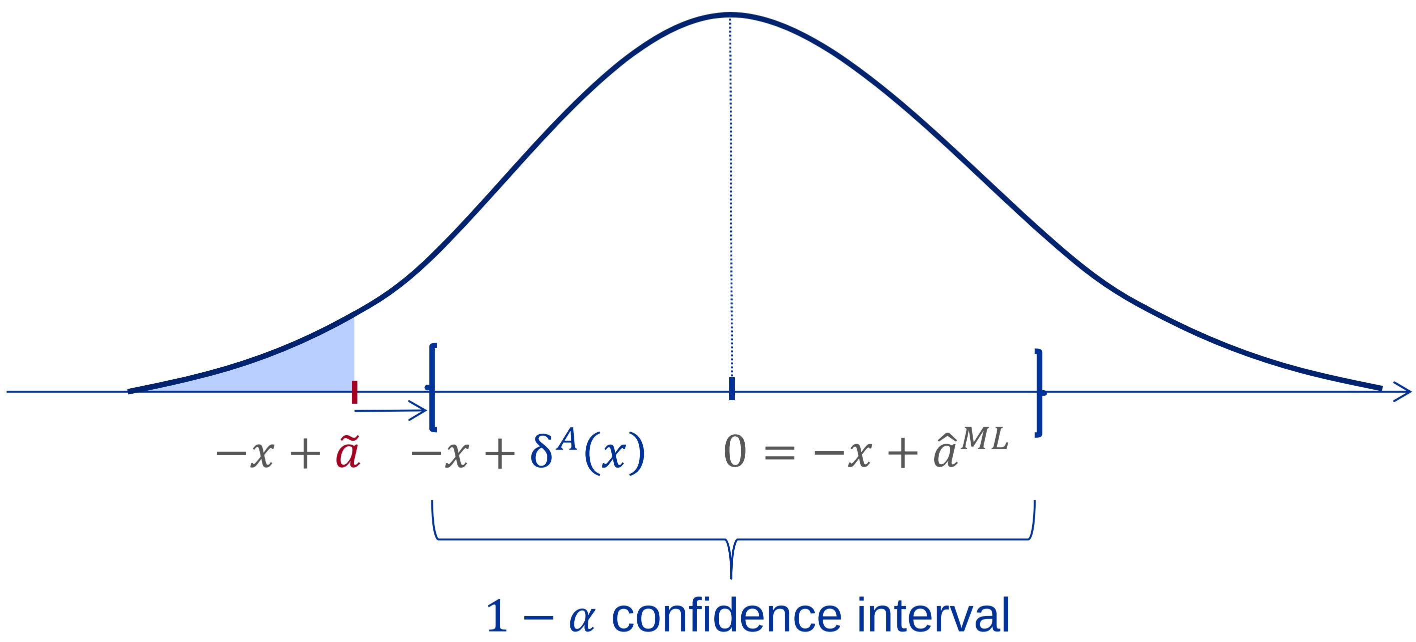
Relationship between p-values and confidence levels

$X \sim N(\theta, 1)$ $\min_a (\theta - a)^2$ $X = x$ Judgment: $\{\tilde{\alpha}, \alpha\}$



p-value $\tilde{\alpha} < \alpha$

Choice of confidence level α

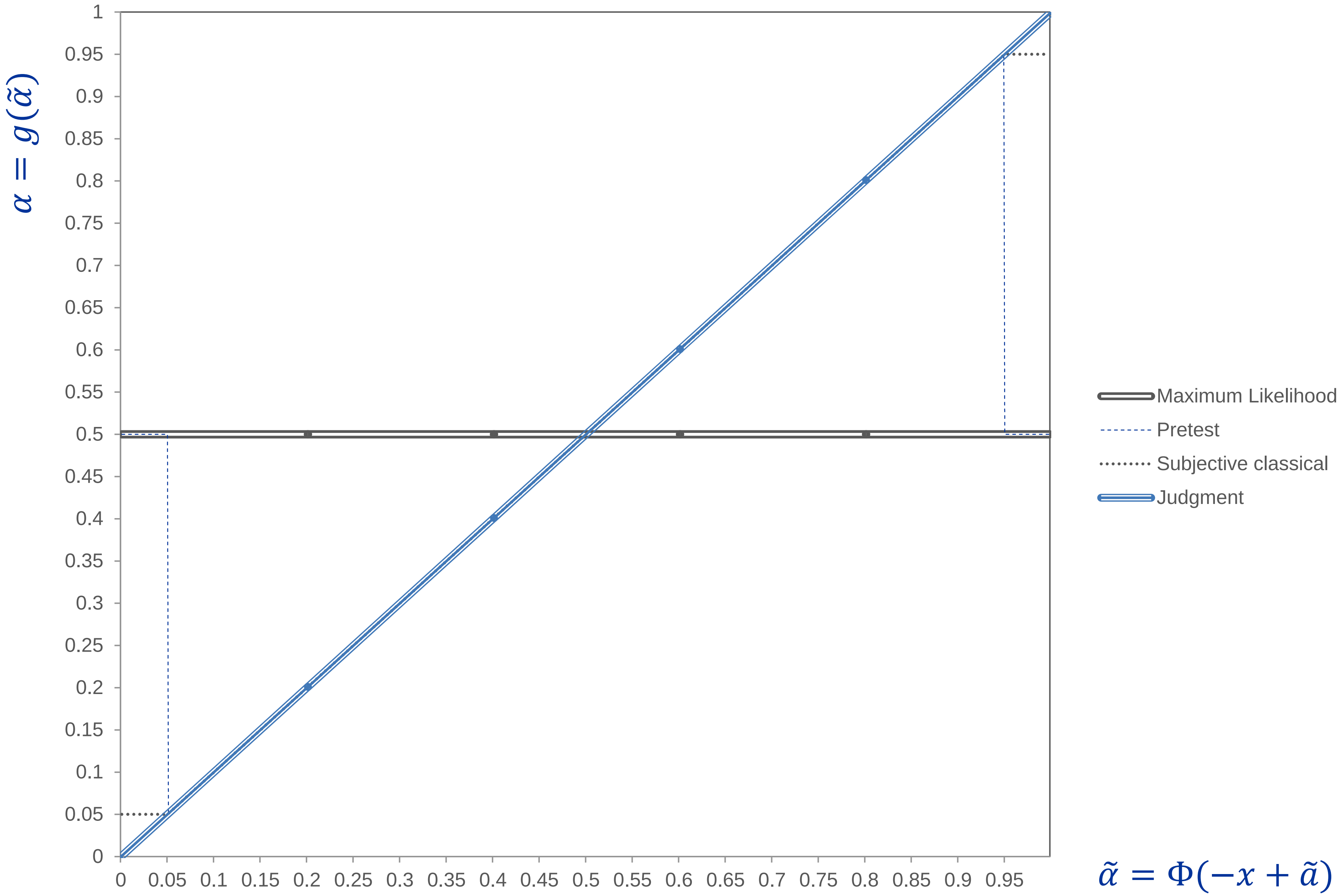
$$\alpha|x = g(\tilde{\alpha}):\ \ (0,1) \rightarrow [0,1]$$

where $\tilde{\alpha} = \Phi(-x + \tilde{a})$

The frequentist decision becomes:

$$\delta^A(x) = x + c_{\alpha|x}$$

Examples of choice of α



Equivalence with Bayesian decision rules

$$\delta^A(x) = x + c_{\alpha|x}$$
$$\Rightarrow \alpha|x = \Phi(\delta^A(x) - x)$$

Impose equivalence condition:

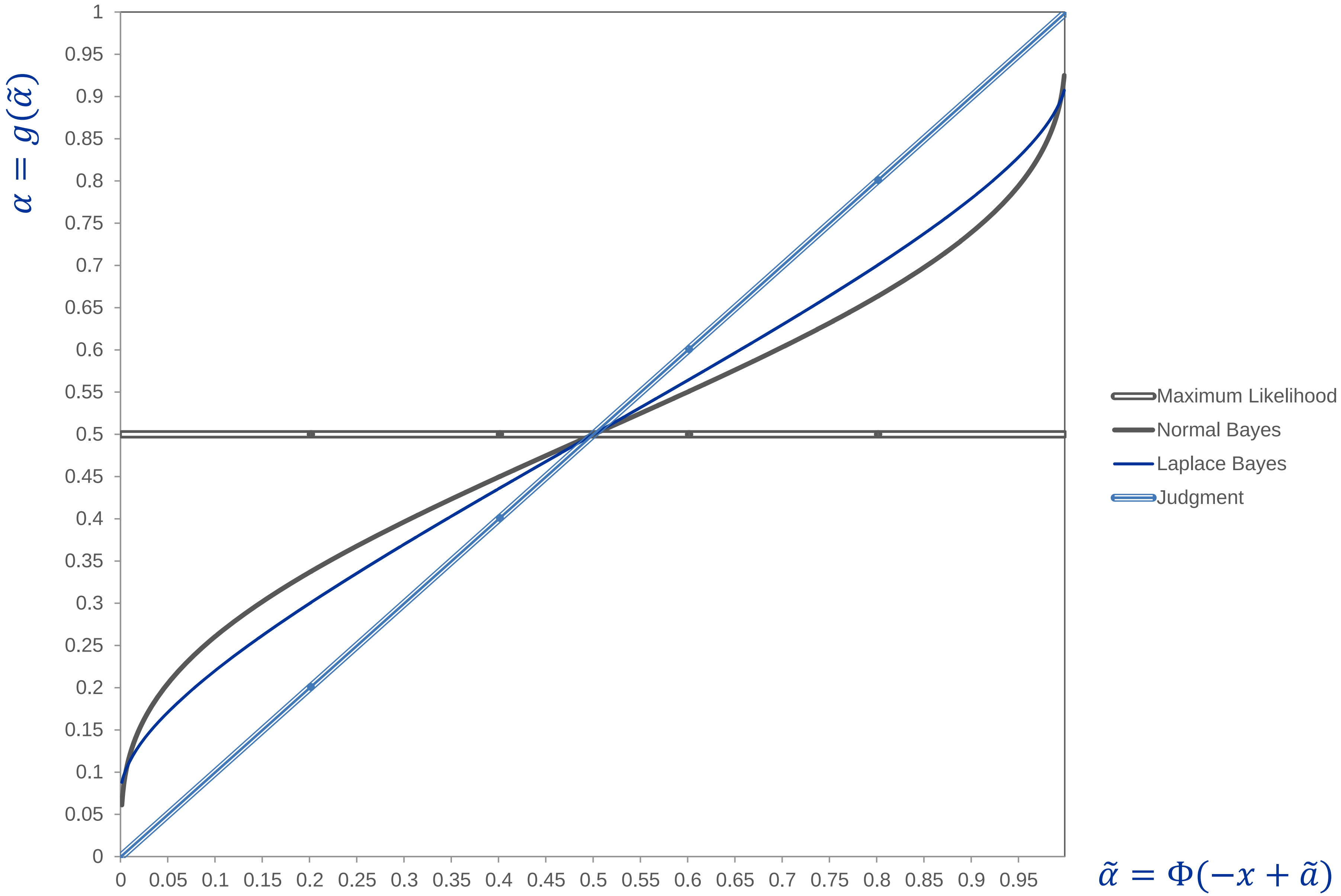
$$\delta^A(x) = \delta^\mu(x)$$

where $x = \tilde{\alpha} - \Phi^{-1}(\tilde{\alpha})$

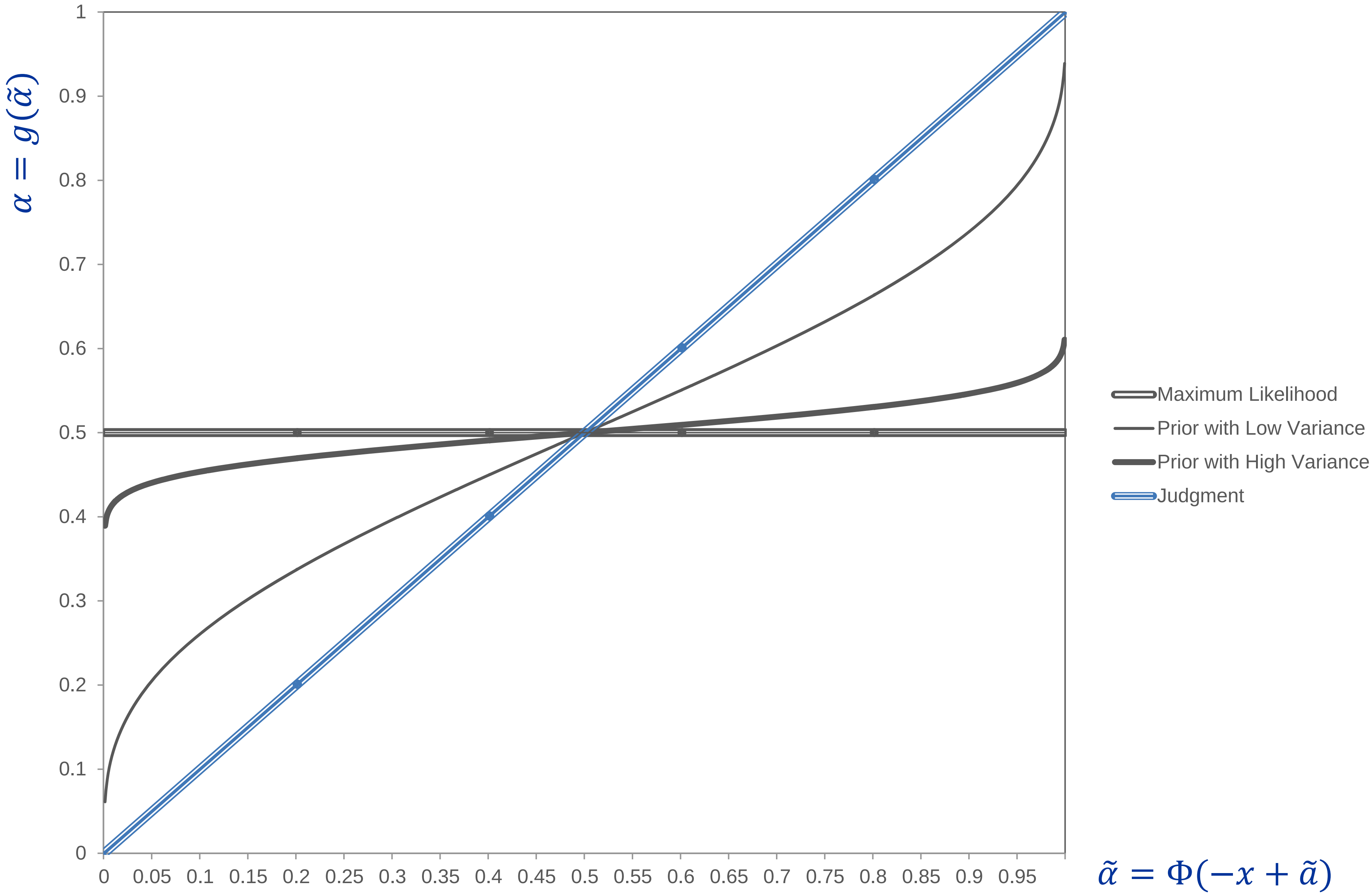
- Normal prior with precision τ:
$$\delta^\mu(x) = x/(1+\tau)$$
- Laplace prior with scale parameter τ:
$$\delta^\mu(x) = x - \tau \frac{1 - \exp\{2\tau x\}\frac{\Phi(-x-c)}{\Phi(x-c)}}{1 + \exp\{2\tau x\}\frac{\Phi(-x-c)}{\Phi(x-c)}}$$

Priors

Bayesian choices of α



Normal priors with difference variances



Outline

SKIP

The asset allocation problem in practice

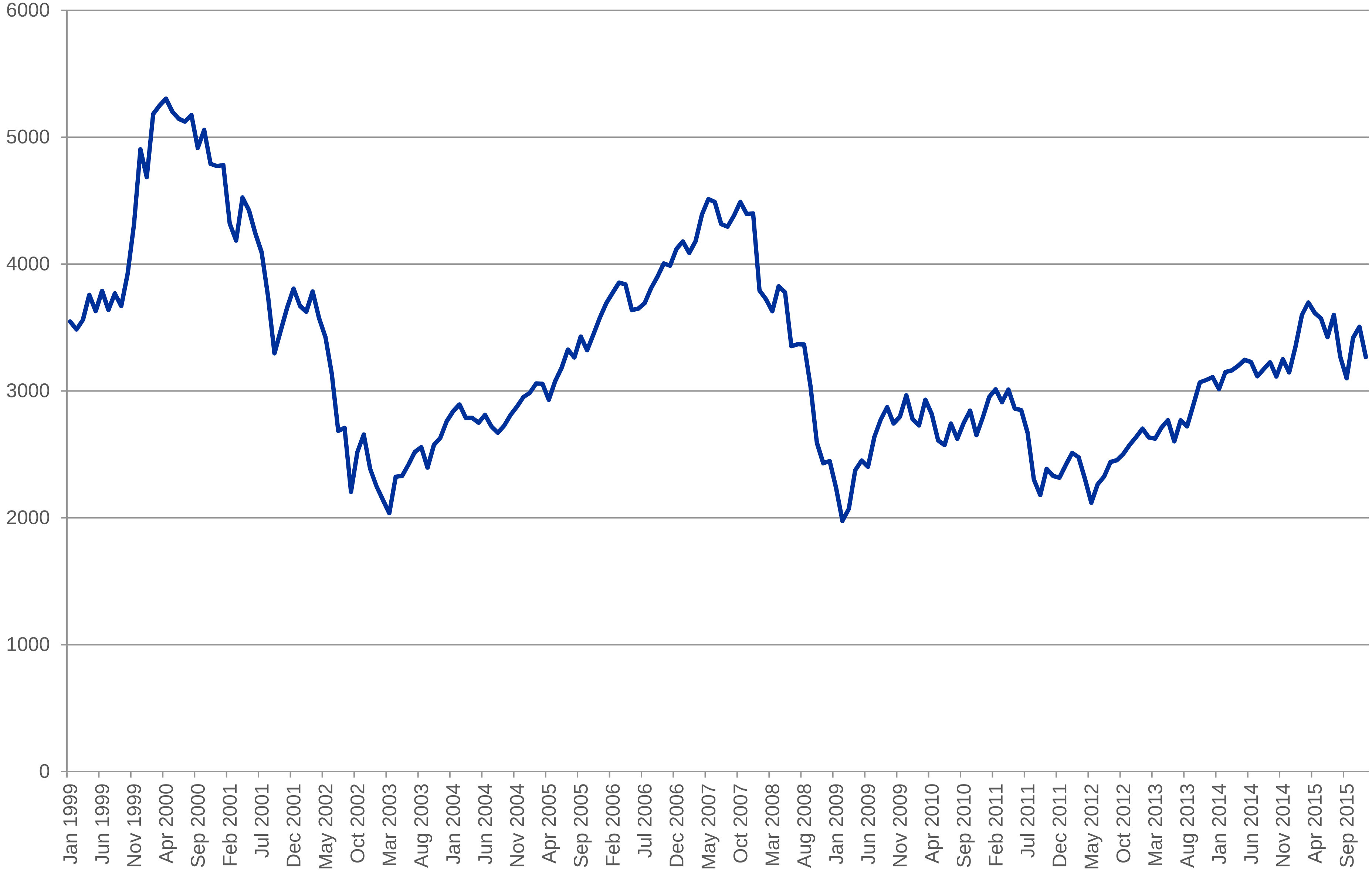
Investor holds cash

Decide how much to invest in EuroStoxx50

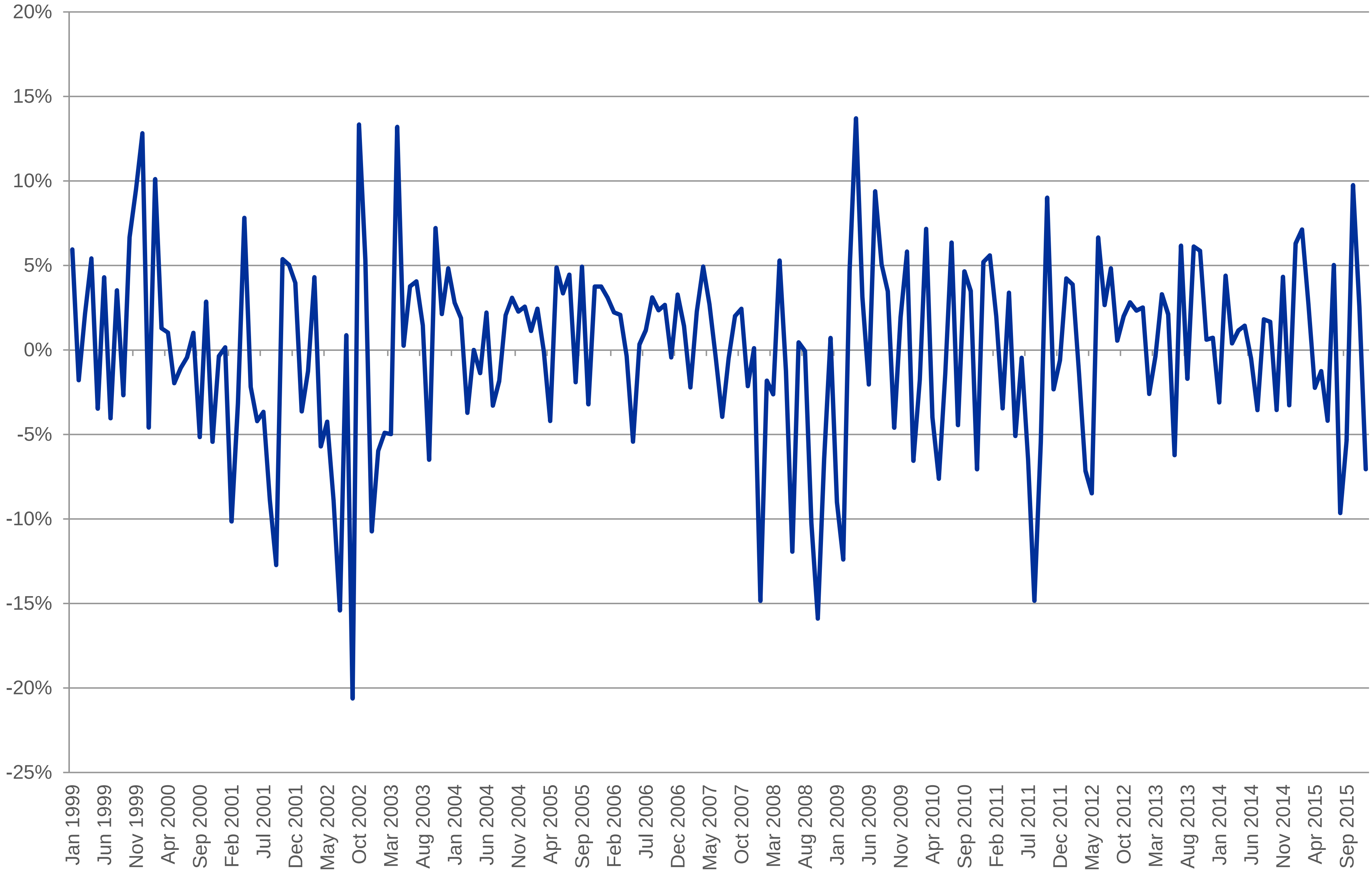
Monthly data from January 1999 until December 2015

Judgmental decision: $\tilde{a} = 0$

EuroStoxx50



EuroStoxx50 – Log returns



Return transformation

To directly apply the estimators discussed so far:

$$\{x_t\}_{t=1,\ldots,T_1+s} = \{\sqrt{(T_1+s)}\tilde{x}_t/\sigma\}_{t=1,\ldots,T_1+s}$$

where:

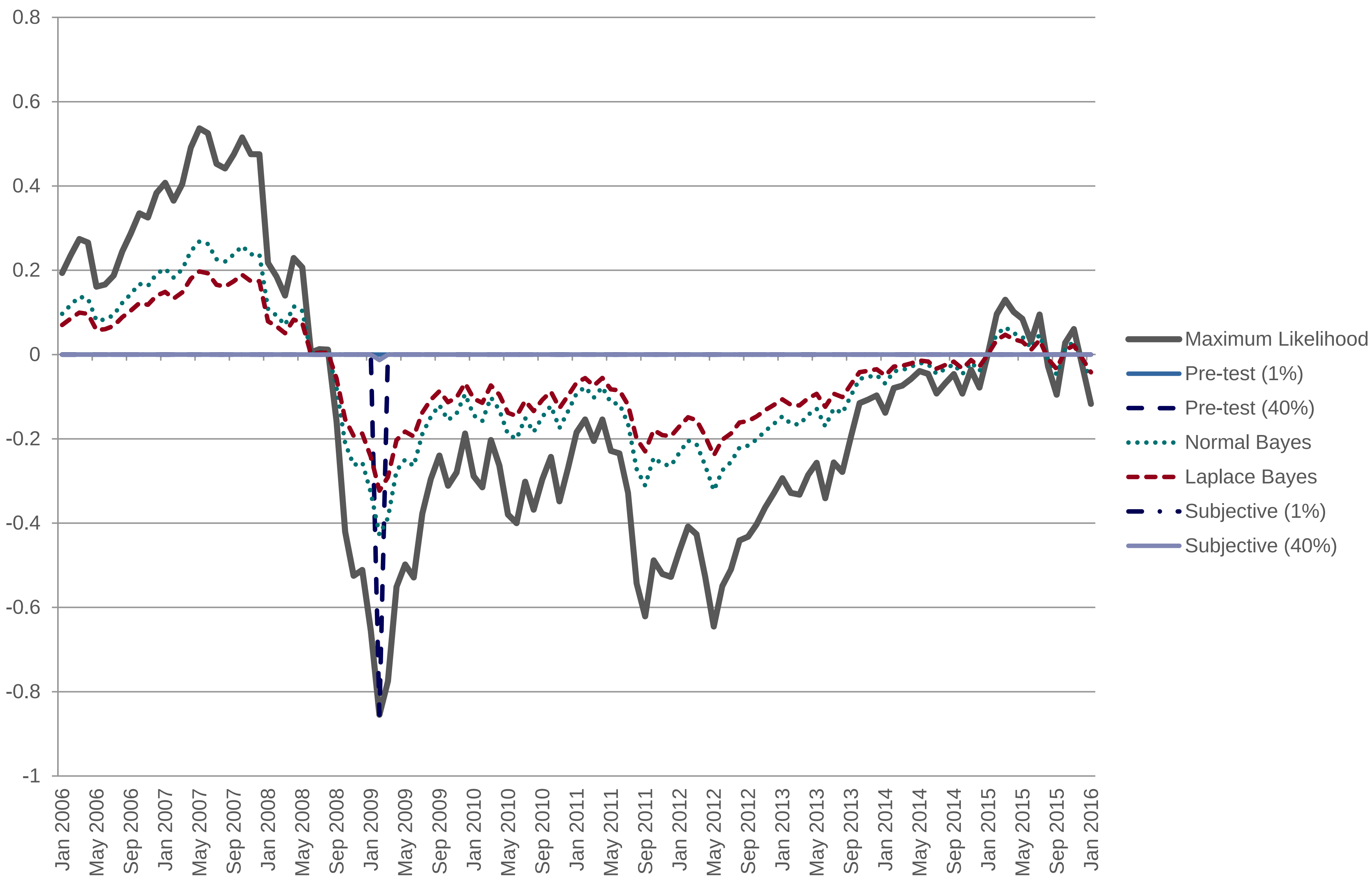
$T_1 = 85$ is the initial sample

σ = full sample standard deviation

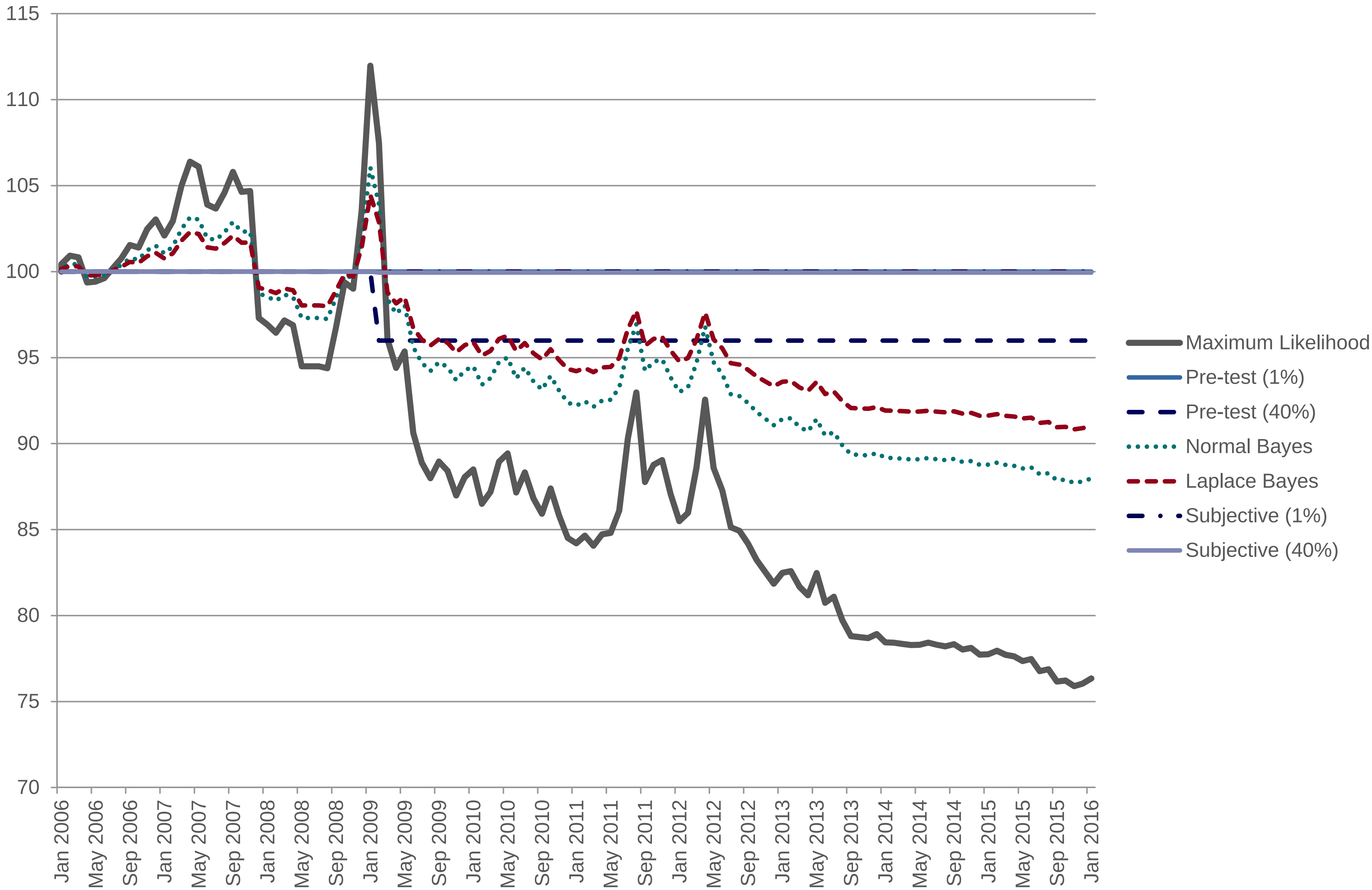
Then:

$$\bar{x}_{T_1+s} = (T_1+s)^{-1}\sum_{t=1}^{T_1+s} x_t \ \sim \ N(\theta^0, 1)$$

Optimal weights



Evolution of portfolio values



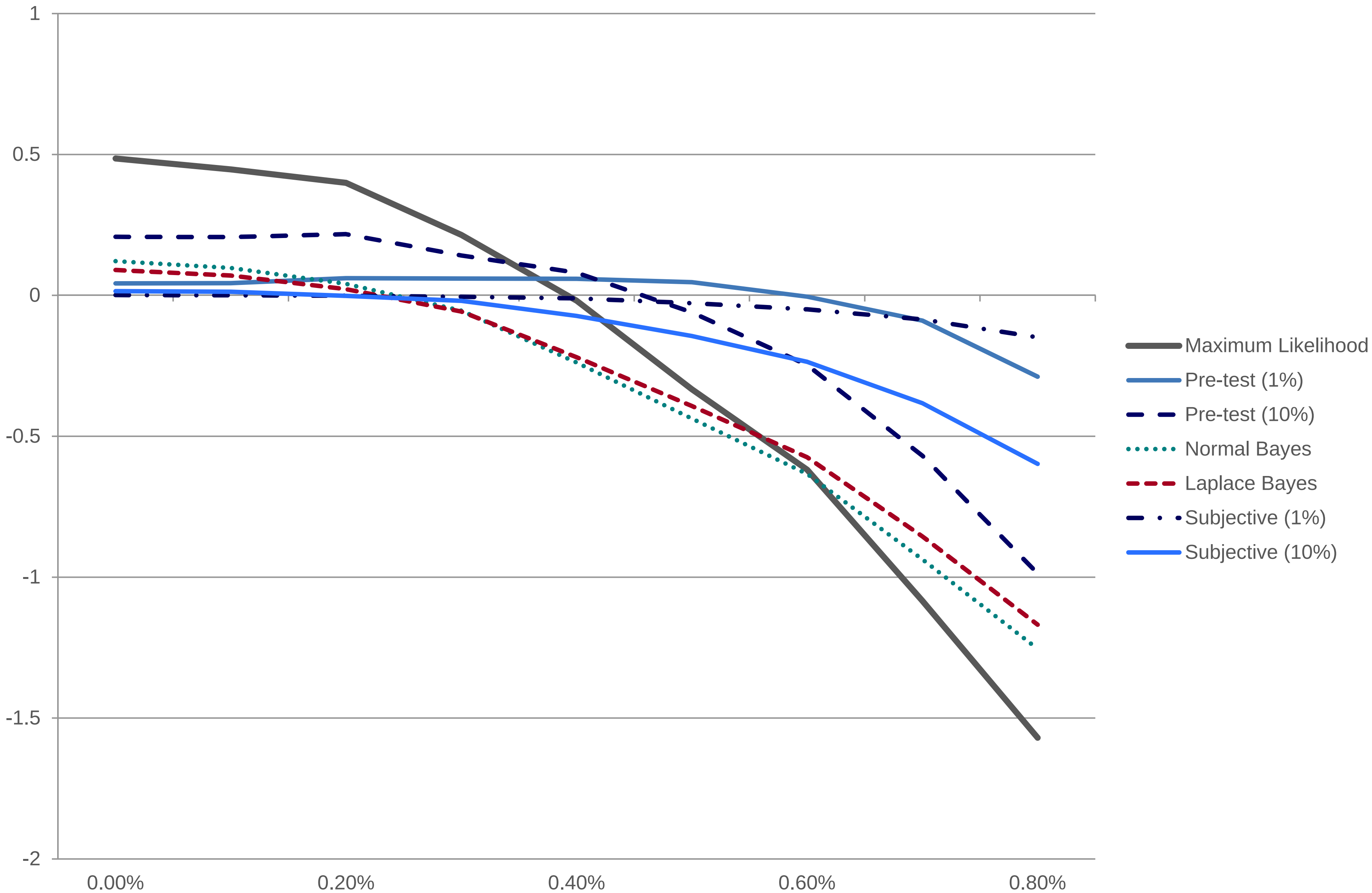
Simulation

1. Generate 500 samples of 206 observations using the demeaned empirical distribution of EuroStoxx50.
2. Generate other sets of 500 samples adding increasing values for the mean

   → the zero judgmental allocation becomes less accurate as the mean of the empirical distribution is increased.

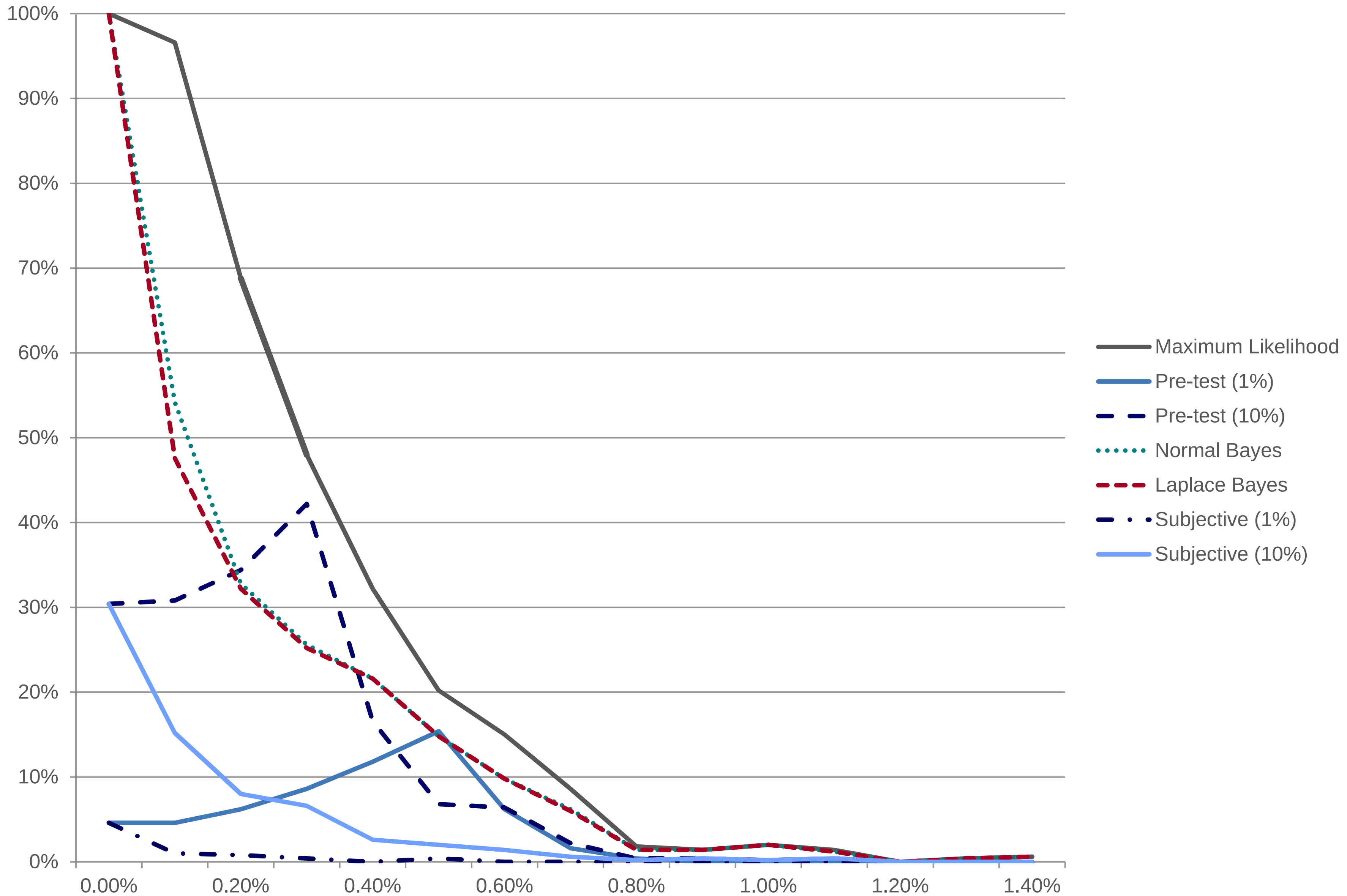
3. Replicate same exercise as before, using first 85 observations for the initial estimation.

Risk



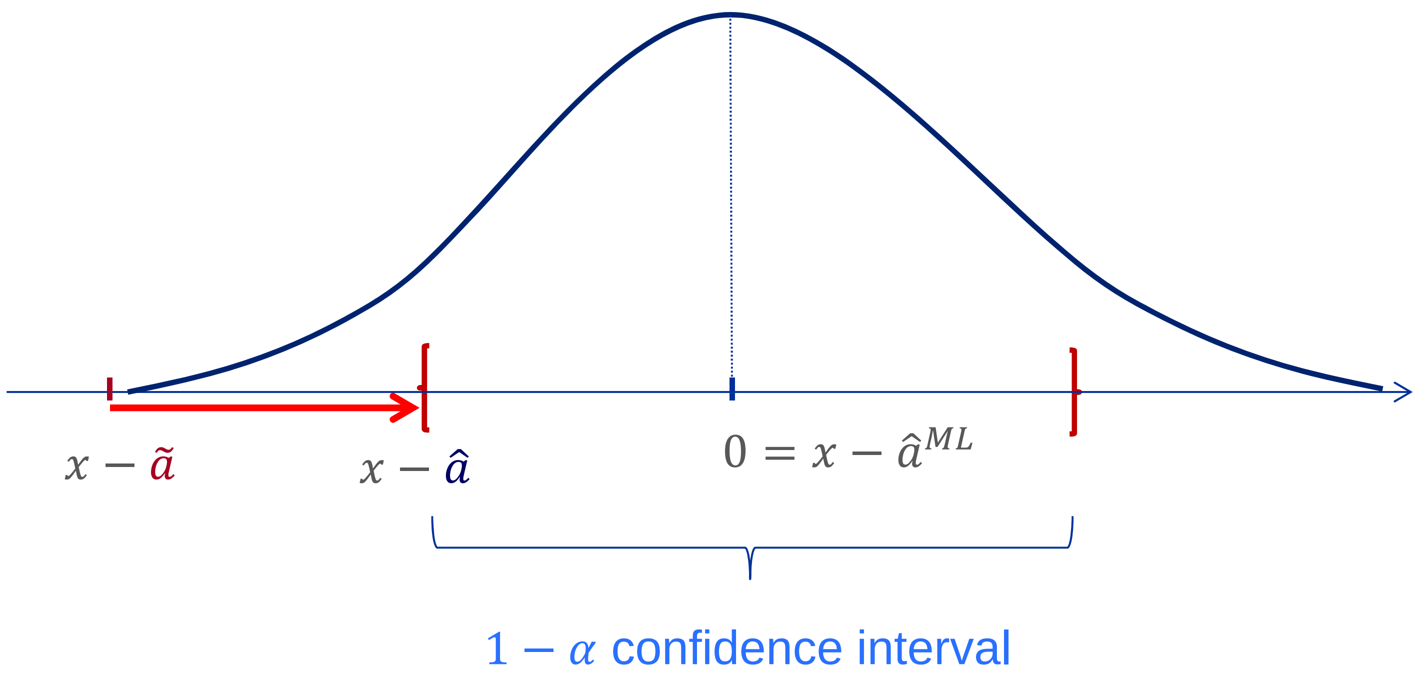
1
0.5
0
-0.5
-1
-1.5
-2
0.00%
0.20%
0.40%
0.60%
0.80%
Maximum Likelihood
Pre-test (1%)
Pre-test (10%)
Normal Bayes
Laplace Bayes
Subjective (1%)
Subjective (10%)

Underperformance relative to judgmental decision



Conclusion

$X \sim N(\theta, 1)$ $\min_a (\theta - a)^2$ $X = x$ Judgment: $\{\tilde{a}, \alpha\}$



Example of prior distributions